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International Refrigeration and Air Conditioning Conference

School of Mechanical Engineering

2004

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CONSTRAINED MULTIOBJECTIVE OPTIMIZATION OF A CONDENSER COIL USING EVOLUTIONARY ALGORITHMS

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ABSTRACT

Air cooled cross-flow heat exchangers are an integral part of refrigeration and air-conditioning systems. The design and selection of a particular heat exchanger depends on its performance and the associated economic parameters, which in turn depend on individual components that make up the heat exchanger. A multiobjective genetic algorithm is applied to the optimization of an air cooled condensing unit. The primary optimization objectives are the performance of the condenser coil and the cost. This study illustrates how genetic optimization algorithms can be a powerful tool to develop optimal designs for air cooled condensers. At the end of the optimization run, the decision maker is presented with a set of Pareto optimal solutions from which the decision maker can choose appropriate solutions. Optimization setup and results are discussed and conclusions drawn.

1 INTRODUCTION

1.1 Background

Applications of air cooled cross-flow heat exchangers include evaporator and condenser coils in an air-conditioning system. Components of each of these heat exchangers include tubes, fins and fans to drive the air flow. The selection of a particular heat exchanger depends on its performance and the associated economic parameters. The performance is dependent on many factors such as the length of tubes, type of tube surface, viz. smooth or enhanced, the type of fin, fin height, width and density, the air flow rate and distribution, fan power, the refrigerant inlet/outlet conditions etc. Economic parameters include material cost, manufacturing cost, cost of fan assembly etc. Clearly the optimization of such a coil is a challenge. There are many different ways to formulate the heat exchanger optimization problem depending upon what performance measures are to be optimized and under what set of constraints. In the current study, an attempt is made to optimize one such air-cooled tube-fin condenser using a multi-objective evolutionary algorithm. The performance of an existing condenser is discussed to serve as a baseline for comparison. The objective is to come up with an optimum fan-coil combination that has the lowest cost and gives the maximum heat rejection capacity and still meets the design constraints. A genetic optimization algorithm (genetic algorithm, GA) is chosen in the light of its abilities to handle complex function optimization and to work with discrete and continuous variables at the same time. The solution methodology is discussed. The results are obtained as a set of Pareto solutions.

1.2 Literature Review

Heat exchanger design optimization has been pursued by many researchers for more than four decades. Fax and Mills (1956) used Lagrange's multiplier approach to optimize a gas turbine regenerator for optimal effectiveness for a given pressure ratio, the pressure ratio itself and the heat exchanger proportions. Hedderich et al.(1982) used an optimization program based on Augmented Lagrangian Multipliers and coupled it with an air-cooled heat exchanger analysis program to carry out constrained single objective (minimum volume for given heat transfer rate) optimization. Chaudhari and Diwekar (1997) used simulated annealing for constrained single objective optimization of heat exchangers. The objectives in their study were minimization of total heat transfer area and the purchase cost. Tayal et al. (1999) demonstrates the first successful application of genetic algorithms to optimize heat exchangers with a black box model. In their study, constrained single objective optimization was performed with two separate objective functions viz. heat exchanger area and purchase cost. The study also compares the performance and results of simulated annealing with genetic algorithms for the same problem. One of the conclusions drawn was that GA's have an advantage over other methods in obtaining multiple solutions of the same quality thus providing more flexibility to the designer.

Genetic Algorithms are relatively new, being first put forth by John Holland in 1975. Since then they have been widely used for single and multiobjective optimization. Some application examples include (Deb, 2001) gas turbine engine design, microprocessor chip design, structural optimization, fishery modeling, medical image reconstruction etc. Fonseca and Fleming (1995) provide an overview of Evolutionary Algorithms for multiobjective optimization. The algorithms are classified on the basis of Pareto based and non-Pareto based approaches. Several other research and application studies are available in literature that include implementation of multiobjective genetic optimization algorithms, their performance comparison etc. The reader is referred to Fonseca and Fleming (1995), Goldberg (1989) and Deb (2001) for further discussion of multiobjective evolutionary algorithms. Some applications of Multiobjective GA's include optimization of building thermal design and solving 2D steady state conduction problems.

2 MULTIOBJECTIVE GENETIC ALGORITHMS

Evolutionary Algorithms are a class of search algorithms based on the principles of natural evolution and include evolutionary programming, evolution strategies, genetic programming and genetic algorithms. Genetic Algorithms as defined by Goldberg (1989) are: "search algorithms based on natural selection and natural genetics"

Genetic Algorithms maintain a pool of candidate solutions, each of which is assigned a fitness based on its usefulness or 'payoff'. Fitness is a measure of how well the particular candidate solution satisfies the given problem objectives and is a scalar value. This payoff value is also termed as Objective Function value. The fitness value may or may not be the same as the payoff or the objective function value. This fitness value determines the directions of search. At each iteration or generation, candidate solutions are selected for reproduction based on their fitness, to form new offspring or solutions. The reproduction process is carried out via the use of genetic operators such as *selection*, *crossover* and *mutation*. A set of probabilistic rules determines whether a candidate solution undergoes *crossover* or *mutation* and at what point. For an excellent introduction to Genetic Algorithms the reader is referred to Goldberg (1989). A powerful feature of GA's is that they search in multiple directions simultaneously and do not require any derivative information or other supplementary information about the problem at hand, only the usefulness or the payoff value. This makes the GA an ideal tool for optimization of highly non-linear functions involving a combination of continuous and discrete design variables.

Real world design optimization problems are seldom characterized by just a single objective. Most of the times there are two or more competing or conflicting objectives that the design engineer has to optimize for and make trade-offs. In addition there is also a set of constraints present. These constraints might be as straightforward as a pair of lower and upper bounds on the design variables or they might be in the form of some equality or inequality that must be satisfied by some non-linear function of the design variables. All these issues make the optimization problem all the more challenging and sometimes difficult.

A Multiobjective optimization problem can be represented mathematically (Deb, 2001) as follows:

$$
\begin{array}{lll}
\text{Minimize/Maximize} & f_m(\mathbf{x}) & m = 1,2\ldots, M \\
\text{such that} & & \\
 & g_j(\mathbf{x}) = 0 & j = 1, 2,\ldots,J \\
 & h_k(\mathbf{x}) = 0 & h = 1,2,\ldots, K \\
 & x_i^L <= x_i <= x_i^U &
\end{array}
$$

where **x** is a vector of n decision variables. The functions g and h are a number of inequality and equality constraints imposed on the design problem. The last line represents the domain constraints for the variable **x**, which restricts each x_i to take a value between x_i^L and x_i^U. If a given solution vector **x** obeys the constraints represented by g_j and h_k, then that solution is termed as a feasible solution, else it is an infeasible solution.

Most multiobjective optimization algorithms use the concept of dominance in their search (Deb, 2001). In order to set the ground for further discussion, some definitions from Deb (2001) are given here. A solution $\mathbf{x}^{(1)}$ is said to dominate the other solution $x^{(2)}$ in a minimization problem, if both conditions a and b are true:

a. The solution $\mathbf{x}^{(1)}$ is no worse than $\mathbf{x}^{(2)}$ in all the objectives, or $f_j(\mathbf{x}^{(1)}) <= f_j(\mathbf{x}^{(2)})$ for all j =1,2,…,M.

b. The solution $\mathbf{x}^{(1)}$ is strictly better than $\mathbf{x}^{(2)}$ in at least one of the objectives, or $f_j(\mathbf{x}^{(1)}) < f_j(\mathbf{x}^{(2)})$ for at least one j belonging to {1,2,…,M}.

Furthermore, a non-dominated set is defined as follows: Among a set of solutions P, the non-dominated set of solutions P\` are those that are not dominated by any member of the set P. When the set P consists of the entire solutions space, then the set P\` is called the Pareto-optimal set. Several algorithms are available in the literature (Deb, 2001) to find the non-dominated set. Thus the goal of a multiobjective optimization algorithm is to find the set of solutions as close to the set of Pareto solutions and at the same time maintain diversity amongst the individual solutions.

Since GA's deal with a scalar fitness value, usually some kind of aggregating method or a utility function is used to assign fitness to solutions. Two fitness assignment schemes implemented and used in this study are described here. The first scheme termed here MOGA1, Nondominated Sorting GA was proposed by Srinivas and Deb (1995). In this scheme, nondominated sorting is performed and the solutions are ranked such that all the solutions in the same nondominated set have the same large fitness value which guarantees every nondominated individual equal reproduction opportunity. Thus the solutions in the first nondominated set/front have the maximum fitness value. The second scheme, termed here as MOGA2, proposed by Fonseca and Fleming (1998) is based on the number of solutions that dominate a particular solution. Thus the fittest solution will have the least rank and thus maximum fitness value.

3 THE OPTIMIZATION PROBLEM

3.1 Objectives and Constraints

The current design objectives were based on the requirements specified by the manufacturer of the coil and comprised of both economic and performance related parameters. The economic parameter considered here is the cost of the coil which is a function of the number of tubes in the condenser, the fins per inch (FPI), number of fans, the width and height of the coil. The performance parameter was the heat rejection capacity of the coil, i.e. for a given mass flow rate through the coil, we seek the maximum heat rejection. It is important to mention here that both objectives are equally important, i.e. this is an example of posteriori preference articulation (Narayanan and Azarm, 1999), wherein the decision maker will be presented with a set of Pareto optimal solutions from which the decision maker can choose.

The constraints are as follows:

a. Refrigerant side pressure drop – The pressure drop on the refrigerant side, which is calculated by the condenser model, should be within specified limits.

b. Fan(s) width vs. tube length - The width of the condenser cabinet is assumed to be equal to the length of the tube. The tube length chosen should be greater than the combined width of the fans placed on the cabinet.
c. Coil height – The height of the coil should be less than the maximum specified value.
d. Tube length – The tube length should be less than or equal to the current tube length, i.e. it cannot be greater than the current tube length.
e. Air side pressure drop – The pressure head supplied by the fans should be the same as the air side pressure drop through the coil. This constraint is handled outside the optimization algorithm, since one can iterate for a matching air flow rate such that the fan pressure drop is the same as the coil pressure drop.
f. Domain constraints – All the independent variables must be within the specified lower and upper bounds where applicable.

The problem is formulated as a two-objective minimization problem with constraints. The first objective is the minimization of negative heat rejection capacity (this essentially maximizes the heat rejection capacity) and the second objective is the minimization of total cost. The optimization algorithm is coupled with an existing simulation tool that predicts the condenser coil performance and cost based on several detailed inputs. The coil performance tool is discussed in section 4.3.

3.2 Design Variables

In this study we try to optimize a condenser such that the condenser built delivers the best performance with minimal cost. The best condenser in this case implies the one that, for a given mass flow rate, gives maximum heat rejection capacity with minimal manufacturing cost and can be assembled from the available components. The available independent variables and their characteristics are summarized as follows:

a. Tube Length (meter) – Continuous Variable – Tube length can be varied between specified lower and upper limits.
b. Tube Outer Diameter – Discrete Variable – 4 different tube sizes are available.
c. Number of parallel circuits – Discrete – The number of parallel circuits in the condenser can have 4 different choices.
d. Number of Fins – Discrete – Expressed in terms of fins per inch, is varied from 6 to 16.
e. Fan Model – Discrete - 20 different fans are available, complete with pressure drop and cost data.
f. Fan Count – Discrete – The number of fans in a coil can vary from 8 to 12, inclusive, in multiples of 2.

4 IMPLEMENTATION

4.1 Multiobjective Genetic Algorithm

The implementation chosen for the current study is a multiobjective genetic algorithm implementation based on Narayanan and Azarm (1999). The implementation includes the MOGA 1 and the MOGA 2 schemes for multi-criterion optimization.

4.2 Algorithm Parameters

The following parameters were used in the algorithm discussed in section 4.1 – population size = 100, crossover probability = 0.9, mutation probability = 0.05, termination criteria = maximum number of iterations. Binary Gray encoding was used, with the number of bits as follows: tube length – 10, parallel circuits (Nt) – 2, Tube diameter (OD) – 2, Fan ID (FID) – 5, FPI – 4, Number of fans (NFan) – 2, etc. A string representation of one of the candidate solution is shown in Figure 1. The decoded values are purposely not included here.

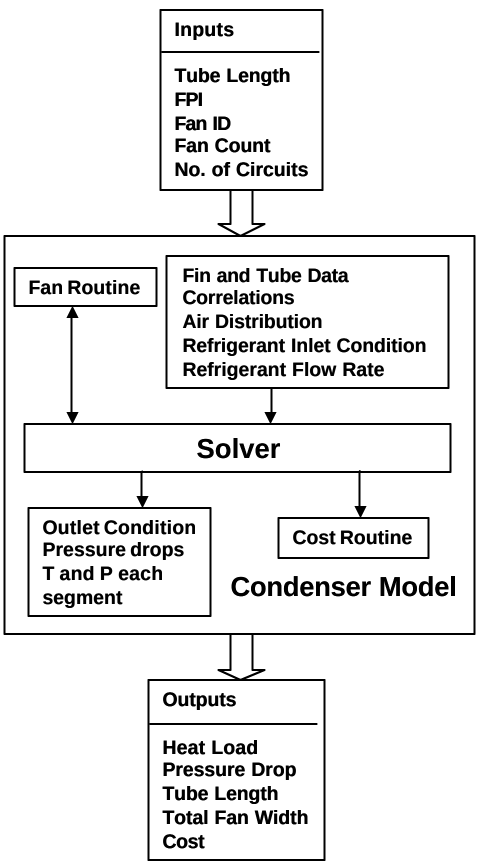
Nt		OD		FID					FPI				NFan		Tube Length									
1	0	0	1	0	1	1	0	0	0	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0

Figure 1: String representation of a candidate solution

Before evaluating the objectives and the constraints the values of design variables obtained from the genetic algorithm are decoded to represent the actual parameter.

4.3 Objective Function - Condenser Model

The condenser model used in the current study is based on the model developed by Jiang et al. (2002) and used in the form of a computer code. The model uses a segmented approach, wherein each tube is divided into a number of segments to account for air flow mal-distribution and refrigerant property variation. The model can account for air velocity variation along the height of the condenser coil in 2 dimensions. The model allows for interactive modification of all the dimensions of the heat exchanger, the number of tubes itself, the fan types, number of fins and the air side parameters such as velocity, relative humidity and temperature. Several refrigerant and air side heat transfer and pressure drop correlations are available in the model for the user to choose from. This model for condenser was modified to match the actual condenser coil performance. This modification was carried out by varying the correction factors for air and refrigerant side heat transfer and pressure drop correlations. The resulting model was considered accurate enough when the outlet sub-cooling matched within ±1K and the heat rejection capacity was within ±4%. The data flow diagram for the condenser model is depicted in Figure 2.



(a)

```
Start
Initialize Condenser Model
Initialize GA
While (GA not done) {
    g = g + 1 "Update generation count"
    Evaluate {
            Decode Variables
            Run Condenser Model
            Calculate constraint violation
            Return (Cost, Heat load) and constraint violation
    }
    Perform Non-Dominated Sorting
    Assign Rank
    Assign Fitness
    Perform Selection, Crossover & Mutation
}
Print Results
End
```

(b)

Figure 2: Flow Diagrams, (a) Condenser model data flow, (b) Optimization Algorithm

4.4 Optimization Setup

A pseudo-code for the entire optimization setup is show in Figure 2b. Note that during each function evaluation, the condenser model is executed which involved first iterating for a matching air flow rate and then solving for the pressures and temperatures.

5 SIMULATION AND RESULTS

The optimization program was executed several times changing the type of fitness ranking procedure and the maximum number of iterations, which is also the termination criterion for the genetic algorithm. The results are summarized in Table 2.

Scheme → Parameter	**MOGA1-250**	**MOGA1-500**	**MOGA2-250**	**MOGA2-500**
Max. Iterations	250	500	250	500
Function Calls	2610	5110	2610	5110
CPU Time(second)	91411	165757	101531	173111
Infeasible Solutions	636	1367	672	1234
Pareto Solutions	93	92	94	92
Better Solution Count	20	44	36	48
Avg. Objective 1	1.0674	1.0745	1.0744	1.0686
Avg. Objective 2	0.9	0.909	0.902	0.896
Max./Min Objective 1	(0.785,1.180)	(0.801,1.181)	(0.804,1.176)	(0.833,1.181)
Max./Min Objective 2	(0.755, 1.131)	(0.761,1.134)	(0.764,1.119)	(0.775,1.133)
Overall Pareto Spread	0.594	0.566	0.528	0.498
Sample Result 1	3,2,16,15,8,0.93	**		
Better Solution = Solution that dominates the baseline case which is (1,1) Objective 1 = Normalized Heat Load, Objective 2 = Normalized Coil Cost For Pareto Spread, Assumed best solution = (1.2,0.7) and bad solution =(0.7,1.2)				

Table 1: Optimization Results Summary

From Table 1, it can be seen that the time required to execute the optimization run seems exorbitantly high. But this time is justified by the fact that the condenser model used is a very detailed one and it requires 30 to 50 second to solve. Also for higher number of iterations, it is seen that the total number of Pareto solutions after the last generation has decreased, but the number of better solutions has increased significantly. Table 1 also shows the number of Pareto optimal solutions at the end of each run as well as the better solution count. Better solution count gives the number of solutions that are better in both the objectives than the existing coil. A quality metric, the overall Pareto spread is also shown in Table 1. The overall Pareto spread metric (Wu and Azarm, 2001) quantifies how widely the observed Pareto solution set spreads over the objective space when all the design objectives are considered together. When comparing multiple Pareto solution sets, the designer prefers the one with a high overall spread, which in the current case is the one obtained by MOGA1-250. Note that approximately 25% of the evaluated solutions in each case were infeasible i.e. one of the constraints was violated.

The values given in Table 2 do not necessarily represent the actual values but are scaled values.

Parameter	**Parallel Circuits**	**Tube Diameter**	**Fan Model**	**FPI**	**Fan count**	**Normalized Tube Length**	**Heat Load**	**Cost**
Result 1	3	2	16	15	8	0.93	1.117	0.984
Result 2	2	2	16	12	8	0.75	1.004	0.846

Table 2: Sample Results for MOGA1, 250 Iterations

Table 2 shows some sample results. For all the Pareto solutions, it was found that the GA chose the least number of fans, a contributing reason for which is the constraint (b) given in section 3.1.

Figure 3a shows the number of Pareto optimal points at each generation for MOGA1-250 iterations. We observe that the initial population itself had 6 Pareto-optimal points. Figures 3b and 3c show the Pareto plots for MOGA1-250 and MOGA1-500 iterations, in which the normalized coil cost versus normalized heat load is plotted. Figure 3d shows a comparative Pareto plot for MOGA2 scheme with 250 and 500 generations. Note that the two Pareto curves almost overlap. From Figure 3b we can conclude that for the same heat rejection capacity the cost can be reduced by 15%.

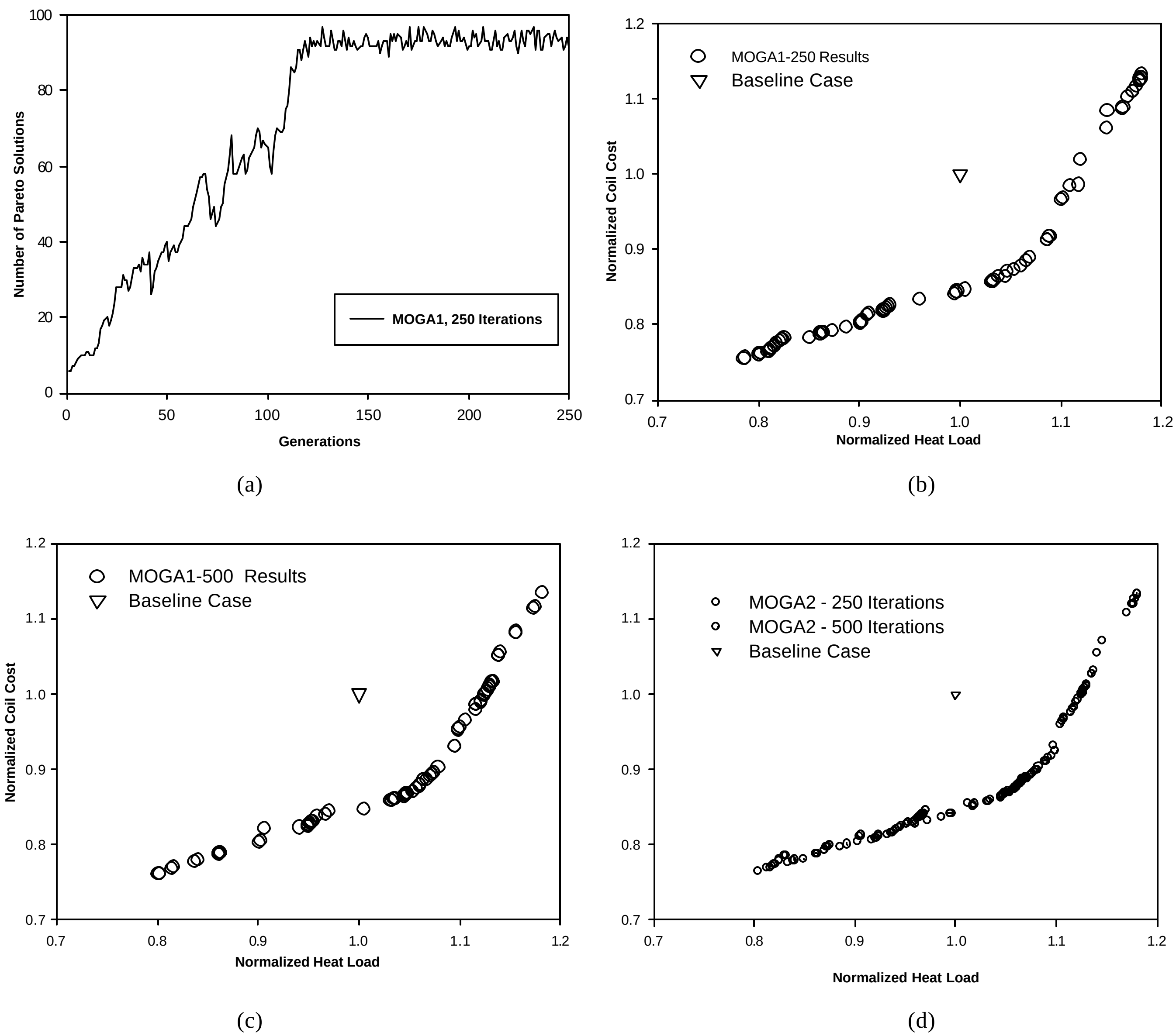


Figure 3: Result Plots, (a) Pareto Solutions Count for MOGA1, 250 Iterations, (b) Pareto Solutions for MOGA1, 250 Iterations, (c) Pareto Solutions, MOGA1, 500 Iterations, (d) Pareto Solutions for MOGA2, 250 and 500 Iterations.

6 CONCLUSIONS

This paper demonstrates the successful application of a multiobjective evolutionary algorithm for the optimization of a tube-fin heat exchanger. The study uses Multiobjective Genetic Optimization Algorithms coupled with a condenser simulation tool to obtain the Pareto-optimal solutions for the heat exchanger cost and the heat rejection capacity. As one would logically expect, the solutions comprised of lesser number of fans (one of the contributing factor to cost) and smaller tube lengths but with increased number of parallel circuits and increased number of fins. The study also uses two different ranking schemes. The results obtained from the two schemes differ in the number of Pareto optimal solutions, but there was no substantial advantage of one over the other for this particular application. Significant computational resources were involved. From an air-conditioning system design point of view one could add an additional constraint that would specify a required sub-cooling at the condenser outlet. The multiple solutions thus obtained, which are of the same quality, provide greater flexibility to the designer, thereby leading to a better system.

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